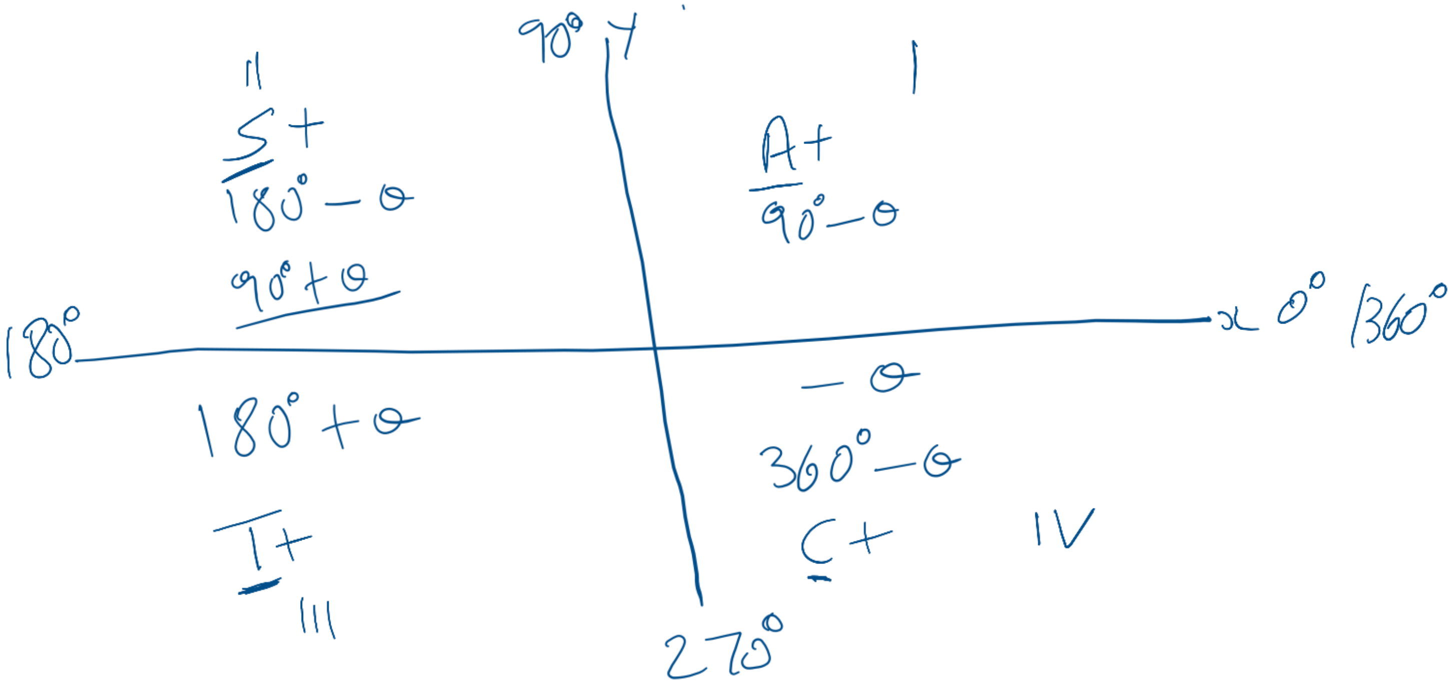
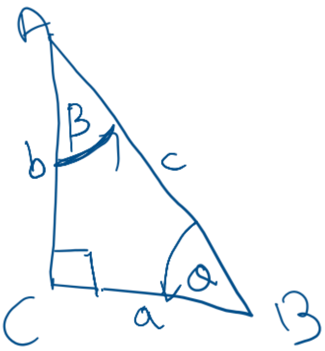
TRIGONOMETRY

Trigonometry Toolbox: Compartment 1

1. Reduction Formulas: CAST Diagram



2. (a) Trigonometric Ratios: Soh Cah Toa



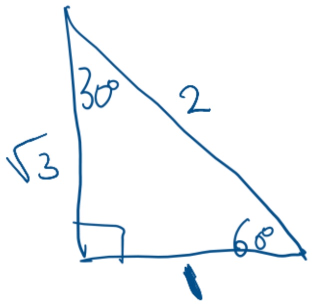
$\sin\theta = \frac{b}{c}$ $\quad\sin\beta = \frac{a}{c}$

$\cos\theta = \frac{a}{c}$ $\quad\cos\beta = \frac{b}{c}$

$\tan\theta = \frac{b}{a}$ $\quad\tan\beta = \frac{a}{b}$

(b) Special angles: $30°, 45° \& 60°$

group 1



$\sin 30° = \frac{1}{2}$

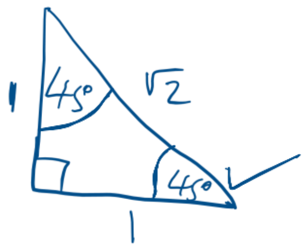
$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$



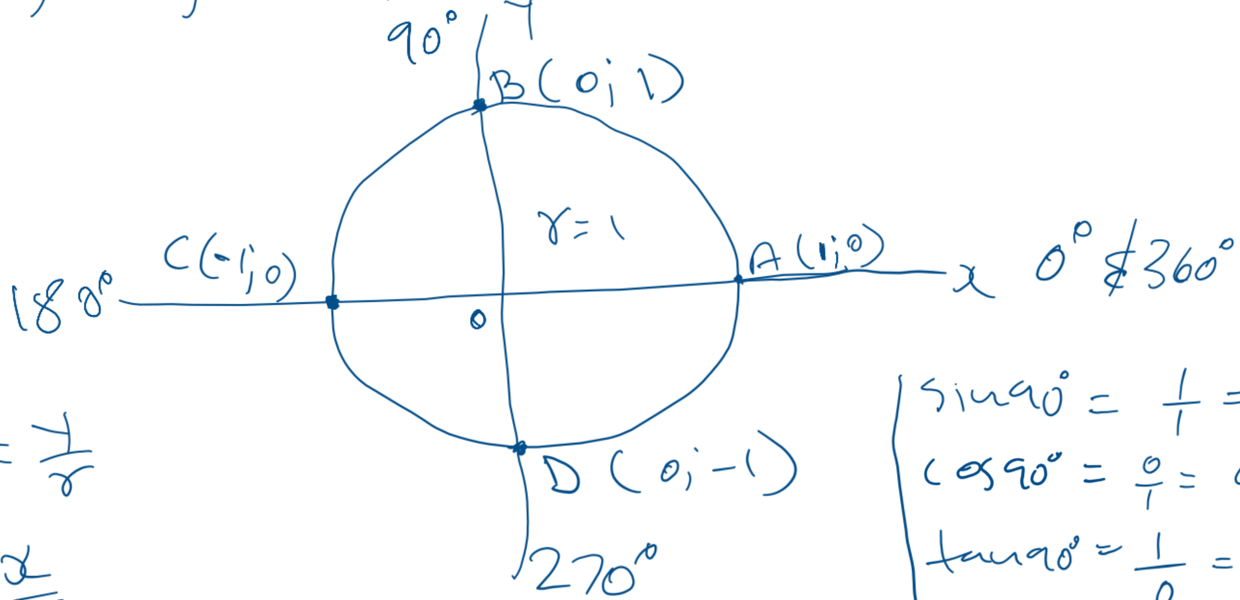
$\sin 45° = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45° = \frac{1}{1} = 1$

Group 2

$0°, 90°, 180°, 270° \& 360°$



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

$\sin 90° = \frac{1}{1} = 1$

$\cos 90° = \frac{0}{1} = 0$

$\tan 90° = \frac{1}{0} =$ undefined

$\sin 180° = \frac{0}{1} = 0$

$\cos 180° = \frac{-1}{1} = -1$

$\tan 180° = \frac{0}{-1} = 0$

3. Identities

- $\sin^2\theta + \cos^2\theta = 1$

  * $\sin^2\theta = 1 - \cos^2\theta$
  * $\cos^2\theta = 1 - \sin^2\theta$

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$

- $\sin 2\theta = 2\sin\theta\cos\theta$

  $\cos 2\theta = \cos^2\theta - \sin^2\theta$

  $= 1 - \sin^2\theta - \sin^2\theta$

  $= 1 - 2\sin^2\theta$

  $\cos 2\theta = \cos^2\theta - \sin^2\theta$

  $= \cos^2\theta - (1 - \cos^2\theta)$

  $= \cos^2\theta - 1 + \cos^2\theta$

  $= 2\cos^2\theta - 1$

- $\sin(\theta + \beta) = \sin\theta\cos\beta + \cos\theta\sin\beta$
- $\cos(\theta + \beta) = \cos\theta\cos\beta - \sin\theta\sin\beta$

$\sin^2(x+1) + \cos^2(x+1) = 1$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$

$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$

Question 1

1.1 Determine**, without the use of the calculator,** the value of the following trigonometric expression:

$$\frac{\sin 2x.\cos(-x)+\cos 2x.\sin(360^\circ - x)}{\sin(180^\circ + x)}$$

$$= \frac{2\sin x\cos x \cdot \cos x + (\cos^2 x - \sin^2 x)\cdot(-\sin x)}{(-\sin x)}$$

$$= \frac{2\sin x\cos^2 x + (-\sin x\cos^2 x + \sin^3 x)}{(-\sin x)}$$

$$= \frac{2\sin x\cos^2 x - \sin x\cos^2 x + \sin^3 x}{(-\sin x)}$$

$$= \frac{\cancel{\sin x}(2\cos^2 x - \cos^2 x + \sin^2 x)}{(-\cancel{\sin x})}$$

$$= \frac{(\cos^2 x + \sin^2 x)}{(-1)}$$

$$= \frac{1}{(-1)}$$

$$= -1$$

(a) $\cos^2 x + \sin^2 x = 1$

(b) $\sin^2 x + \cos^2 x = 1$

N.B

(a) $\cos^2 x - \sin^2 x = \cos 2x$

(b) $\sin^2 x - \cos^2 x \neq \cos 2x$

$$\sin^2 x - \cos^2 x = -\cos^2 x + \sin^2 x$$
$$= -(\cos^2 x - \sin^2 x)$$
$$= -(\cos 2x)$$
$$= -\cos 2x$$

1.2 Prove that $\cos(90° - 2x).\tan(180° + x) + \sin^2(360° - x) = 3\sin^2 x$

$L.H.S = \cos(90° - 2x).\tan(180° + x) + \sin^2(360° - x)$

$= \sin 2x . \tan x + [\sin(360° - x)]^2$

$= 2\sin x \cancel{\cos x} . \frac{\sin x}{\cancel{\cos x}} + [-\sin x]^2$

$= 2\sin^2 x + \sin^2 x$

$= 3\sin^2 x$

$= R.H.S$

1.3 **Without the use of a calculator**, prove that:

a) $\frac{\tan 480°.\sin 300°.\cos 14°.\sin(-135°)}{\sin 104°.\cos 225°} = \frac{3}{2}$.

$L.H.S = \frac{\tan 480°.\sin 300°.\cos 14°.\sin(-135°)}{\sin 104°.\cos 225°}$

$= \frac{\tan 120°.\sin(360° - 60°).\cos 14°.\sin 225°}{\sin(180° - 76°).\cos(180° + 45°)}$

$= \frac{\tan(180° - 60°).(-\sin 60°).\cos 14°.\sin(180° + 45°)}{\sin 76°.(-\cos 45°)}$

$= \frac{(-\tan 60°).\left(-\frac{\sqrt{3}}{2}\right).\cancel{\sin 76°}.(-\sin 45°)}{\cancel{\sin 76°}.\left(-\frac{\sqrt{2}}{2}\right)}$

$= \frac{(-\sqrt{3}).\left(-\frac{\sqrt{3}}{2}\right).\cancel{\left(-\frac{\sqrt{2}}{2}\right)}}{\cancel{\left(-\frac{\sqrt{2}}{2}\right)}}$

$= \frac{3}{2}$

$= R.H.S$

b) $\cos 75° = \frac{\sqrt{2}(\sqrt{3}-1)}{4}$

$$L.H.S = \cos 75°$$
$$= \cos(30° + 45°)$$
$$= \cos 30° \cos 45° - \sin 30°.\sin 45°$$
$$= \frac{\sqrt{3}}{2}.\frac{\sqrt{2}}{2} - \frac{1}{2}.\frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{3}.\sqrt{2}}{4} - \frac{\sqrt{2}}{4}$$
$$= \frac{\sqrt{3}.\sqrt{2} - \sqrt{2}}{4}$$
$$= \frac{\sqrt{2}(\sqrt{3}-1)}{4}$$
$$= R.H.S$$

1.4 **Without using a calculator**, determine the value of: $\frac{\sin^2 35° - \cos^2 35°}{4\sin 10° \cos 10°}$

$$\frac{\sin^2 35° - \cos^2 35°}{4\sin 10°.\cos 10°}$$
$$= \frac{-\cos 2.35°}{2.2\sin 10° \cos 10°}$$
$$= \frac{-\cos 70°}{2.\sin 2.10°}$$
$$= \frac{-\cos 70°}{2\sin 20°}$$
$$= \frac{-\cancel{\sin 20°}}{2\cancel{\sin 20°}} = -\frac{1}{2}$$

$\cos^2\theta - \sin^2\theta = \cos 2\theta$

$\sin^2\theta - \cos^2\theta = -\cos 2\theta$

$2\sin\theta\cos\theta = \sin 2\theta$

1.5 **Simplify** $\sin(180° - x).\cos(-x) + \cos(90° + x).\cos(x - 180°)$ **to a single trigonometric ratio.**

$\sin(180° - x) . \cos(-x) + \cos(90° + x) . \cos(x - 180°)$

$= \sin x . \cos x + (-\sin x) . \cos(180° + x)$

$= \sin x \cos x + (-\sin x) . (-\cos x)$

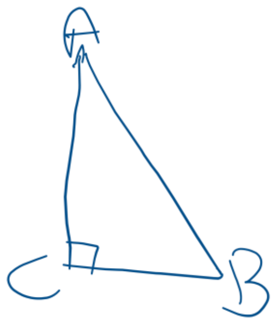
$= \sin x \cos x + \sin x \cos x$

$= 2 \sin x \cos x$

$= \sin 2x$

Trigonometry Toolbox: Compartment 2

- Theorem of pythagoras



$$AB^2 = CB^2 + AC^2$$

$$AB^2 - AC^2 = CB^2$$

$$\sqrt{CB^2} = \sqrt{AB^2 - AC^2}$$

$$CB = \sqrt{AB^2 - AC^2}$$

- $\sin^2\theta + \cos^2\theta = 1$; θ is acute

$$\sqrt{\sin^2\theta} = \sqrt{1 - \cos^2\theta}$$

$$\sin\theta = \sqrt{1 - \cos^2\theta}$$

Question 2

2.1 If $\sin 24° = p$, express the following in terms of p, **without the use of a calculator**:

$\sin 24° = \frac{p}{1}\ \frac{o}{h}$

2.1.1 $\cos 24°$

$AB^2 = AC^2 + CB^2$ (pythagoras)

$(1)^2 = (p)^2 + CB^2$

$1 = p^2 + CB^2$

$\sqrt{CB^2} = \sqrt{1-p^2}$

$CB = \sqrt{1-p^2}$

$\cos 24° = \frac{\sqrt{1-p^2}}{1} = \sqrt{1-p^2}$

$\sin^2 24° + \cos^2 24° = 1$

$\sqrt{\cos^2 24°} = \sqrt{1-\sin^2 24°}$

$\cos 24° = \sqrt{1-\sin^2 24°}$

$= \sqrt{1-(p)^2}$

$\cos 24° = \sqrt{1-p^2}$

2.1.2 $\sin 12° \cos 12° - \sin(-66°)\tan 204°$

$2\sin\theta\cos\theta = \sin 2\theta$

$= \frac{2}{2}\cdot\sin 12°\cdot\cos 12° - (-\sin 66°)\cdot\tan(180°+24°)$

$= \frac{1}{2}\cdot 2\sin 12°\cdot\cos 12° + \sin 66°\cdot\tan 24°$

$= \frac{1}{2}\cdot\sin 2.12° + \frac{\cancel{\sqrt{1-p^2}}}{1}\cdot\frac{p}{\cancel{\sqrt{1-p^2}}}$

$= \frac{1}{2}\cdot\sin 24° + p$

$= \frac{1}{2}\cdot p + p$

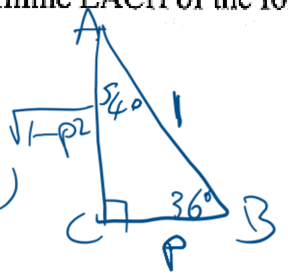
$= \frac{3}{2}p$

2.2 Given: $\sin 36° = \sqrt{1-p^2}$ $\frac{o}{h}$

$\frac{\sqrt{1-p^2}}{1}$

Without using a calculator, determine EACH of the following in terms of p:

2.2.1 $tan36°$



$AB^2 = CB^2 + AC^2$ (pythagoras)

$(1)^2 = CB^2 + (\sqrt{1-p^2})^2$

$1 = CB^2 + 1 - p^2$

$p^2 = CB^2$

$\sqrt{CB^2} = \sqrt{p^2}$

$CB = p$

$\tan 36° = \frac{\sqrt{1-p^2}}{p}$

2.2.2 $cos108°$

$\cos 108° = \cos(180° - 72°)$

$= -\cos 72°$

$= -\cos 2.36°$

$= -(1 - 2\sin^2 36°)$

$= -(1 - 2.(\sqrt{1-p^2})^2)$

$= -[1 - 2.(1-p^2)]$

$= -[1 - 2 + 2p^2]$

$= -[-1 + 2p^2]$

$= 1 - 2p^2$

$= -2p^2 + 1$

2.3 Given: $\cos 26° = m$ $\frac{a}{h}$

Without using a calculator, determine $2\sin^2 77°$ in terms of m.

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2.13° = 2\cos^2 13 - 1$

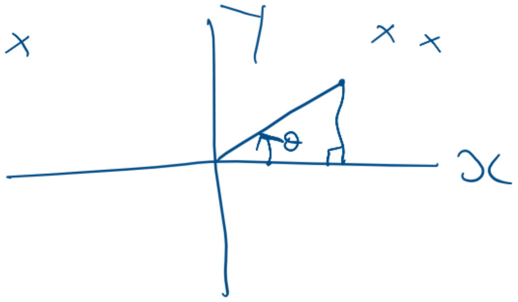
$\cos 26° + 1 = 2\cos^2 13$

$$2\sin^2 77° = 2(\sin 77°)^2$$
$$= 2.(\cos 13°)^2$$
$$= 2\cos^2 13°$$
$$= \cos 26° + 1$$
$$= m + 1$$

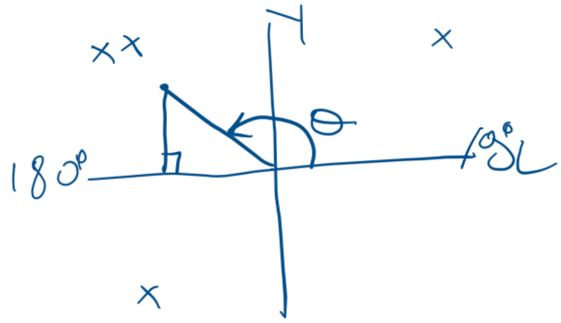
Trigonometry Toolbox: Compartment 3

→ Drawing △ in the correct quadrant.

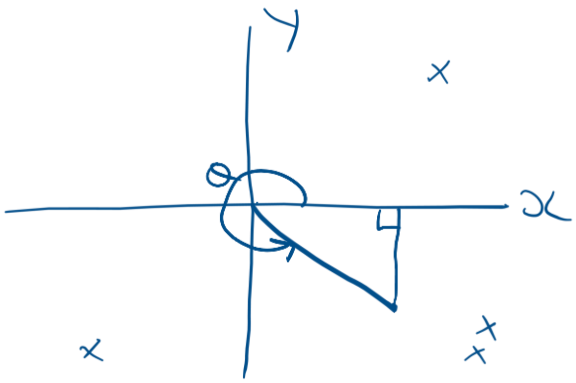
(a) $\sin\theta = \frac{1}{2}$ where $\theta \in [0°; 90°]$



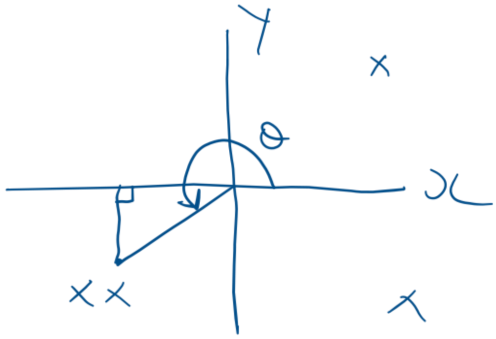
(b) $\cos\theta = -\frac{1}{2}$ where $\theta \in [0°; 180°]$



(c) $\sin\theta = -\frac{1}{4}$ where $\cos\theta > 0$



(d) $\tan\theta = \frac{5}{2}$ and $\sin\theta < 0$



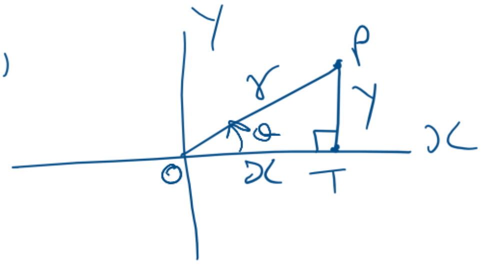
N.B $3\tan\theta - 5 = 7$ where

$\frac{3\tan\theta}{3} = \frac{12}{3}$

$\tan\theta = 4$

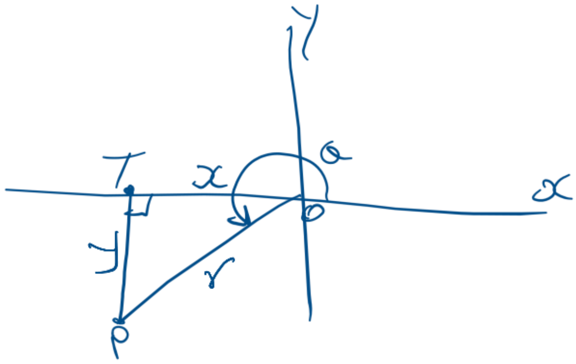
- Pythagoras in terms of x^2, y^2 & r^2

(a)



$r^2 = x^2 + y^2$ (pythagoras)

(b)



$r^2 = x^2 + y^2$ (pythagoras)

Question 3

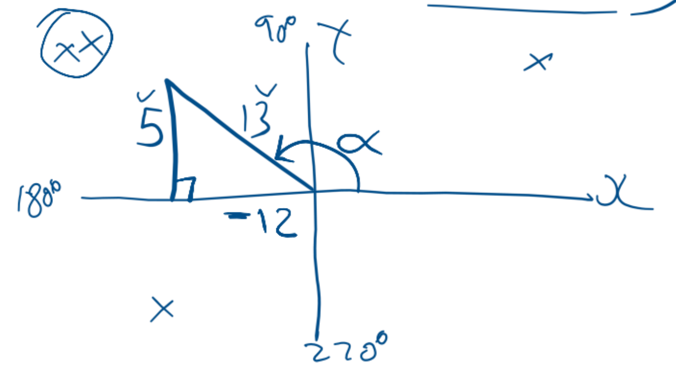
3.1 It is known that $13\sin\alpha - 5 = 0$ and $\tan\beta = -\frac{3}{4}$ where $\alpha \in [90°; 270°]$ and $\beta \in [90°; 270°]$. Determine, without using a calculator, the values of the following:

3.1.1 $cos\alpha$

$13\sin\alpha - 5 = 0$ where $\alpha \in [90°; 270°]$

$13\sin\alpha = 5$

$\sin\alpha = \frac{5}{13}$ $\frac{y}{r}$



$r^2 = x^2 + y^2$

$(13)^2 = x^2 + (5)^2$

$169 = x^2 + 25$

$\sqrt{x^2} = \sqrt{144}$

$x = \pm 12$ $\therefore x = -12$

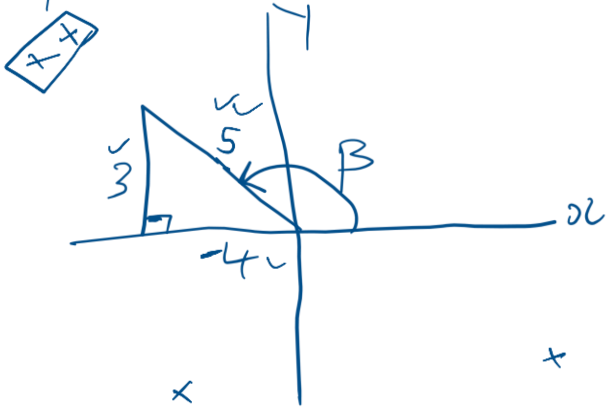
$\therefore \cos\alpha = \frac{x}{r}$

$= \frac{-12}{13}$

$= -\frac{12}{13}$

3.1.2 $\cos(\alpha + \beta)$ $= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\tan\beta = -\frac{3}{4}$ $\frac{y}{x}$ where $\beta \in [90°; 270°]$



$r^2 = x^2 + y^2$ (pythagoras)

$r^2 = (-4)^2 + (3)^2$

$\sqrt{r^2} = \sqrt{25}$

$r = 5$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

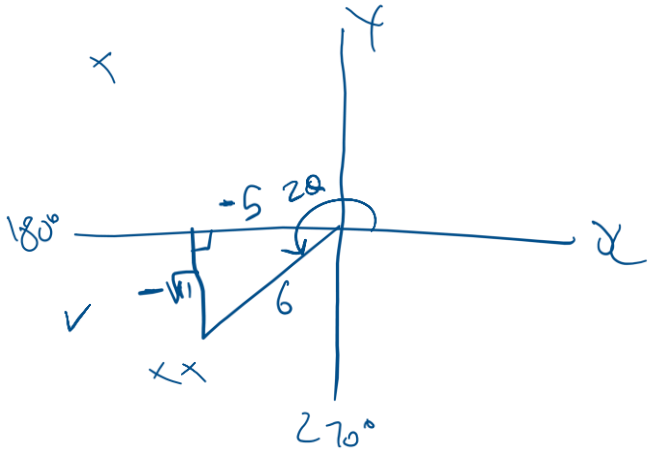
$$= \left(\frac{-12}{13}\right)\times\left(\frac{-4}{5}\right) - \left(\frac{5}{13}\right)\times\left(\frac{3}{5}\right)$$

$$= \frac{48}{65} - \frac{15}{65}$$

$$= \frac{33}{65}$$

3.2 If $\cos 2\theta = -\frac{5}{6}$, where $2\theta \in [180°; 270°]$, calculate, **without using a calculator,** the values in simplest form of:

3.2.1 $sin2\theta$



$r^2 = x^2 + y^2$ (pythagoras)

$(6)^2 = (-5)^2 + y^2$

$36 = 25 + y^2$

$\sqrt{y^2} = \sqrt{11}$

$y = \pm\sqrt{11}$ $\therefore y = -\sqrt{11}$

$\sin 2\theta = \frac{y}{r} = -\frac{\sqrt{11}}{6} = -\frac{\sqrt{11}}{6}$

3.2.2 $\sin^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta - \sin^2\theta = \cos 2\theta$

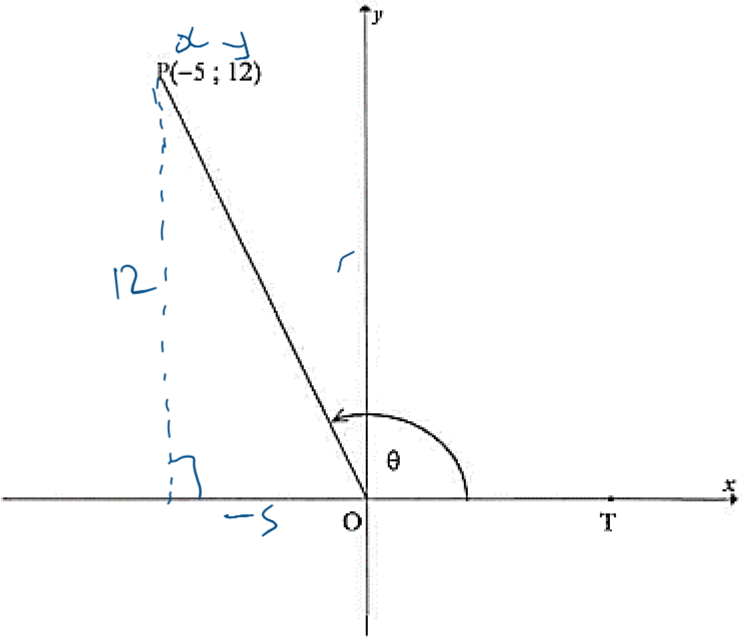
$1 - \sin^2\theta - \sin^2\theta = -\frac{5}{6}$

$1 - 2\sin^2\theta = -\frac{5}{6}$

$-2\sin^2\theta = -\frac{11}{6}$

$\sin^2\theta = \frac{11}{12}$

3.3 In the diagram, P(–5 ; 12) and T lies on the positive x-axis. $P\hat{O}T = \theta$



Answer the following **without using a calculator:**

3.3.1 Write down the value of $tan\theta$.

$\tan\theta = \frac{y}{x} = -\frac{12}{5}$

3.3.2 Calculate the value of $cos\theta$.

$r^2 = x^2 + y^2$ (pythagoras)

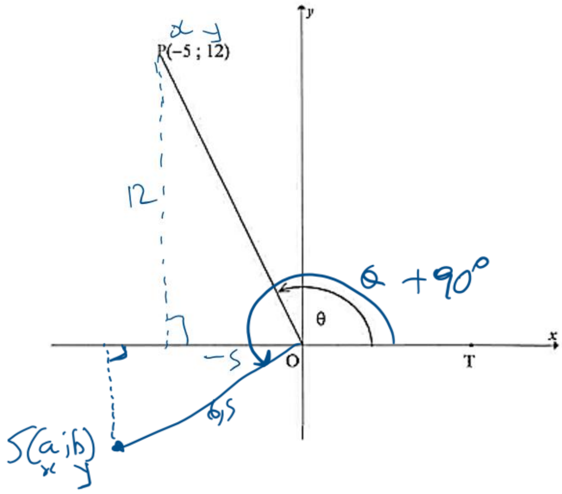
$r^2 = (-5)^2 + (12)^2$

$\sqrt{r^2} = \sqrt{169}$

$r = 13$

$\cos\theta = \frac{x}{r} = \frac{-5}{13} = -\frac{5}{13}$

3.3.3 S(a ; b) is a point in the third quadrant such that $T\hat{O}S = \theta + 90°$ and OS = 6,5 units. Calculate the value of b.



$$\sin(\theta + 90°) = \frac{b}{6,5}$$

$$\sin(90° + \theta) = \frac{b}{6,5}$$

$$\cos\theta = \frac{b}{6,5}$$

$$-\frac{5}{13} = \frac{b}{6,5}$$

$$-\frac{5}{2} = b$$

$$\therefore b = -\frac{5}{2}$$